Strukturpålitelighet og miljøkonturer

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03 February 2016

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Outline

- Introduction and background
  - Probabilistic structural design; First Order Reliability Method (FORM)
- Traditional approach to environmental contours
  - FORM-based; Rosenblatt transformation
  - Convexity properties
- Alternative approach
  - Based on direct Monte Carlo simulations
  - Algorithm for identifying the boundary
  - Three specific estimation methods
  - Importance sampling technique
- Comparison study
- Results and discussions

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Introduction and background

- Extreme environmental conditions impose extreme loads and stresses on marine structures and may lead to structural failure
- Load and response calculations need to be used in marine design to ensure adequate structural strength
  - What are the operating conditions the structure can be expected to encounter throughout its lifetime?
- In principle, long-term response analyses required for each design alternative
  - Time and computational intensive

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Introduction

- Environmental contours often applied in marine structural design
- Allows for considering environmental loads independent of designs
  – Identifying design sea states applicable to all designs
  – Time consuming load calculations only for a limited set of design sea states
- Often based on IFORM
  – Rosenblatt transformation to standard normal space
- Examples presented in two dimensions
  – Easily extended to higher dimensions (in principle)

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Joint environmental model

- Need a description of the environmental conditions
  - simultaneous distribution of several environmental parameters
  - E.g. Significant wave height (Hs) and mean wave period (Tz)
- Environmental contours will be based on the assumed joint environmental model
- Often, a conditional modelling approach is used
  - Fit a marginal distribution to the primary environmental variable, e.g. Hs
  - Conditional distributions of secondary environmental variables, e.g. Tz
- Other joint models may also be used, e.g. based on copulas

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Probabilistic structural design

- Assume some stochastic input variables $\mathbf{X} = (X_1, X_2, ..., X_n)^T$
  - With joint density function $f_{\mathbf{X}}(\mathbf{x})$
- Assume a performance function, g(**X**), only dependent on **X**
  - g(**X**) > 0: Structure survives
  - g(**X**) < 0: Structure fails
  - g(**X**) = 0 is the limit state function: boundary between safe and unsafe regions of the **X**-space
- Reliability, *R*, of the structure

$$R = 1 - P_f = P[g(\boldsymbol{X}) > 0] = \int_{g(\boldsymbol{x})>0} f_X(\boldsymbol{x})\, d\boldsymbol{x}$$

  - Reliability integrals normally difficult to solve exactly as both g(x) and f(x) might be complicated functions → FORM and SORM approximations

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First Order Reliability Method (FORM)

- Transform **X** to **U** = $(U_1, U_2, \ldots, U_n)^T$ using the Rosenblatt transformation, then:

$$R = 1 - P_f = P[\tilde{g}(\boldsymbol{U}) > 0] = \int_{\tilde{g}(\boldsymbol{u})>0} \phi(\boldsymbol{u}) d\boldsymbol{u}$$

  – with $\tilde{g}(\boldsymbol{u})$ - transformed performance function

- Approximate the failure boundary at the design point by a first order Taylor expansion

  – Reliability index, β_r: Distance from design point to the origin

$$R = 1 - P_f \approx \Phi(\beta_r)$$

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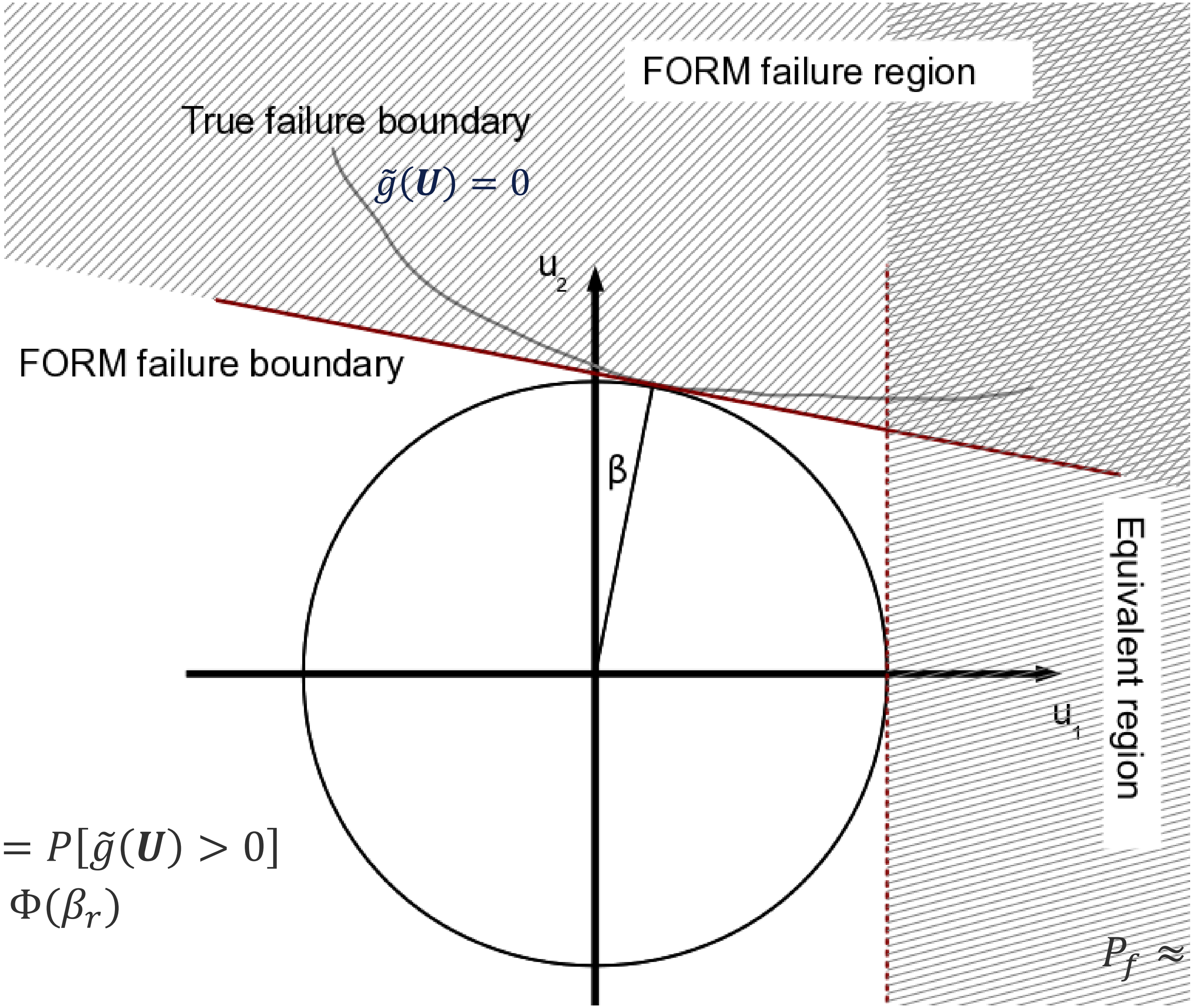
The FORM approximation in U-space



$$R = 1 - P_f = P[\tilde{g}(\boldsymbol{U}) > 0]$$
$$\approx \Phi(\beta_r)$$

$$P_f \approx 1 - \Phi(\beta_r)$$

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Definition used in this presentation

- Let **X** be a vector of environmental variables with possible values in the set $\mathcal{X}$. Let P_f be a given failure probability
- The objective is to identify a convex set $\mathcal{B} \subset \mathcal{X}$ such that for every tangent plane Π of the set $\mathcal{B}$ we have $P[\boldsymbol{X} \in \Pi^+] = P_f$ where Π^+ denotes the halfspace bounded by the plane Π and not containing $\mathcal{B}$
- The resulting environmental contour is the boundary of the set $\mathcal{B}$, denoted $\partial\mathcal{B}$
- Hence, for any design with convex failure region $\mathcal{F}$ such that $\mathcal{F} \cap \mathcal{B} = \emptyset$, the failure probability will be less than P_f

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Traditional approach

- Transform $\boldsymbol{X}$ into a vector $\boldsymbol{U}$ of standard normal variables (Rosenblatt transformation)
- Let $\tilde{\mathcal{B}}$ be an n-dimensional sphere centered around the origin with radius β_r, where $\Phi(\beta_r) = 1 - P_f$
  - Obtain a set with the desired properties in $\boldsymbol{U}$-space
- Transform the set back to the original environmental parameter space to obtain the set $\mathcal{B}$ (inverse Rosenblatt transformation)
- $\mathcal{B}$ will not necessarily have the desired properties:
  - That is, if Π is a tangent plane of this set and Π^+ denotes the halfspace bounded by the plane Π and not containing $\mathcal{B}$, then there are no guarantee that $P[\boldsymbol{X} \in \Pi^+] = P_f$
  - May occur since the Rosenblatt transformation is generally non-linear

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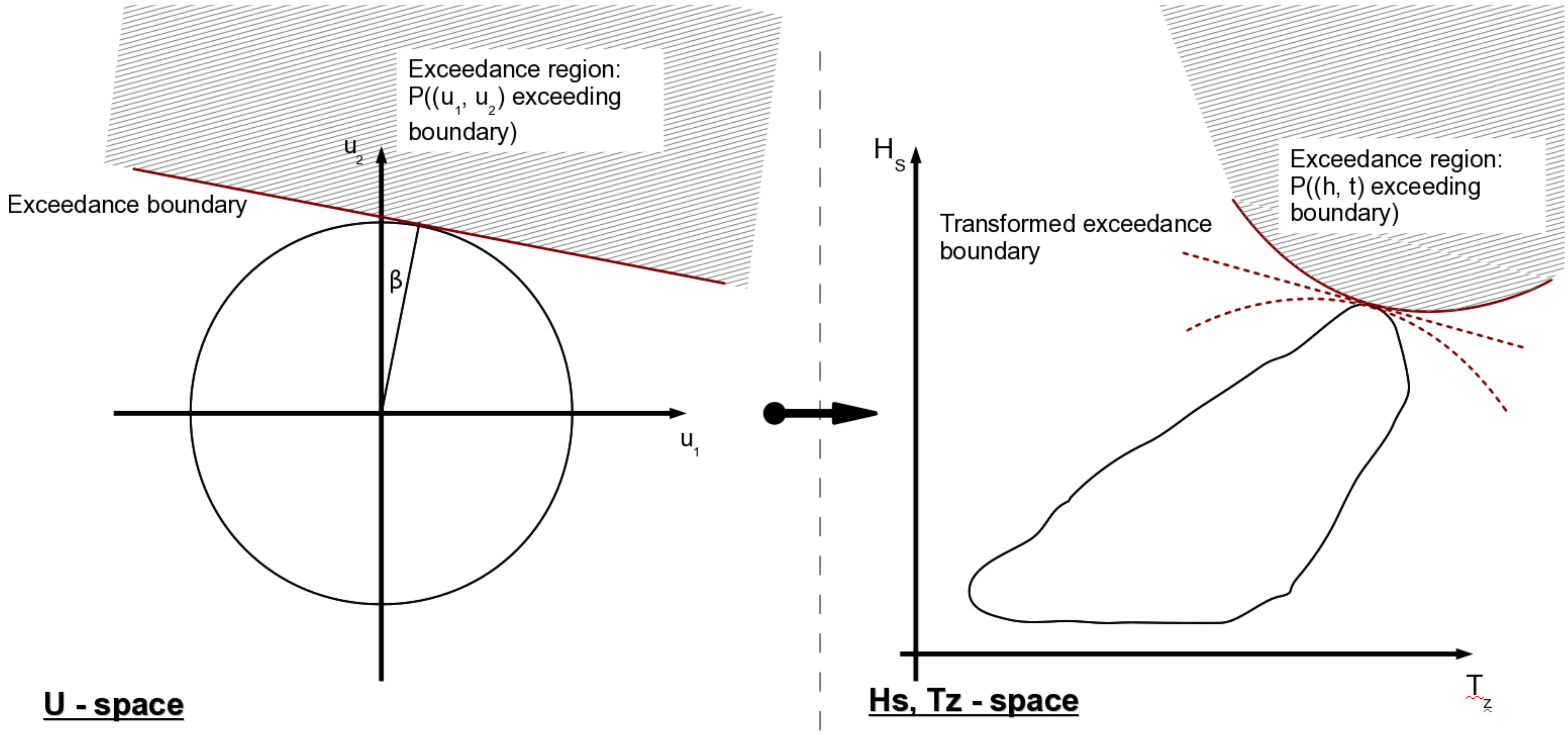
Convexity properties of traditional contours



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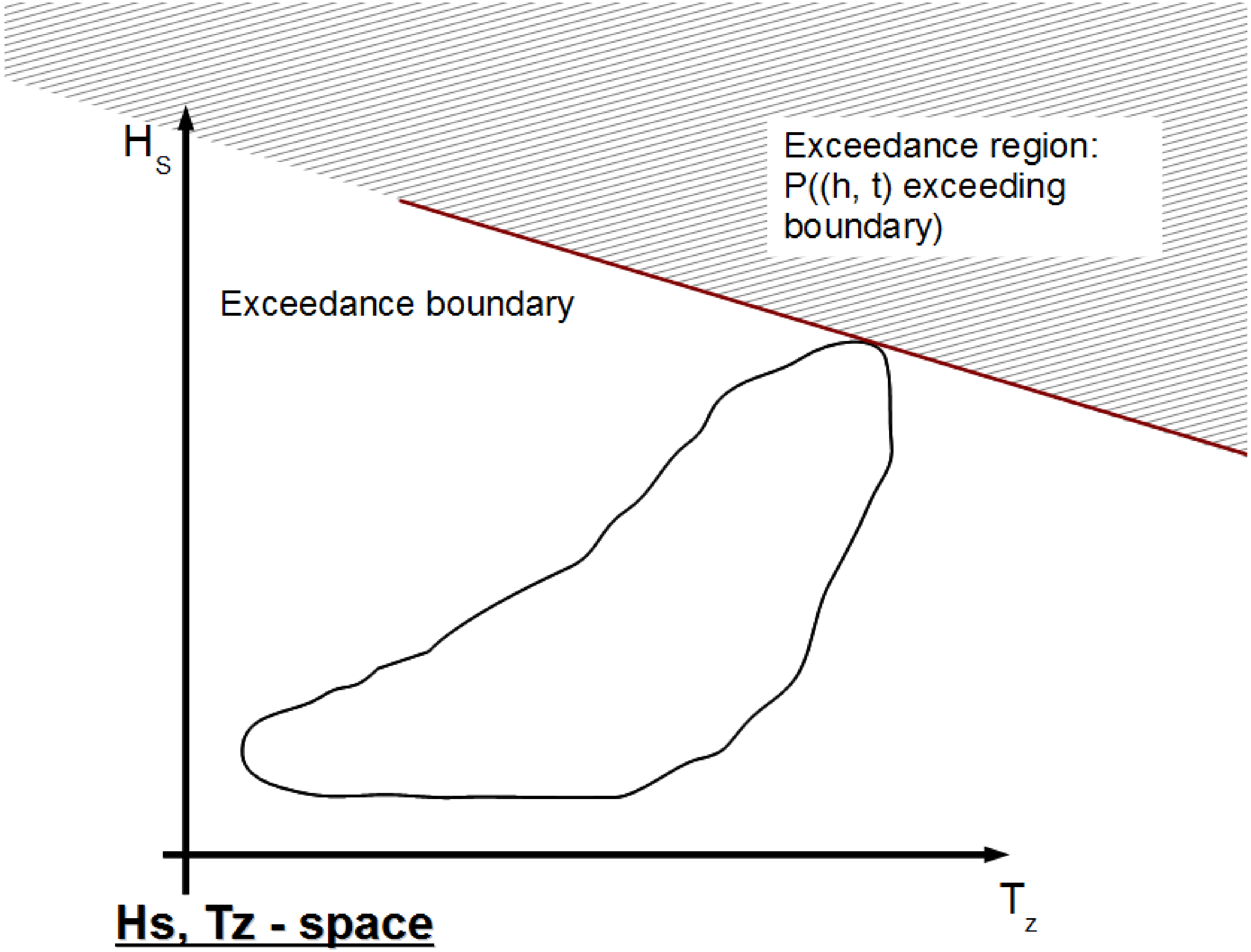
Failure region in alternative environmental contours



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Obtaining contours by direct Monte Carlo simulations

- Let P_f be the required failure probability
- Simulate from the joint environmental model $f(h,t)$
- Then, for any given angle, $\theta \in [0,360)$, identify a straight line $\Pi(\theta)$ defined by $t\cos\theta + h\sin\theta = C(\theta)$ partitioning the space in two halfspaces $\Pi(\theta)^+$ and $\Pi(\theta)^-$so that the fraction of samples in $\Pi(\theta)^+$ is approximately P_f
- The set $\mathcal{B}$ is then the intersection of all sets $\Pi(\theta)^-$ for $\theta \in [0,360)$

$$\mathcal{B} = \bigcap_{\theta \in [0,360)} \Pi(\theta)^-$$

- Need to identify the function $C(\theta)$ - the distance from the origin to the tangent line - for selected angles θ

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Generic method

1. Simulate n points $(T_1, H_1), \dots, (T_n, H_n)$
2. Calculate the projections at angle θ,
$$X_i = T_i \cos\theta + H_i \sin\theta, \qquad i = 1, \dots, n$$
3. Sort in ascending order: $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$ with corresponding samples $\left(T_{(1)}, H_{(1)}\right), \dots, \left(T_{(n)}, H_{(n)}\right)$
4. Calculate number of samples to be kept within the desired failure boundary: $k = n(1 - P_f)$
5. Identify the halfspace, for each angle θ,
$$\mathcal{B}(\theta) = \left\{(t, h)\colon\, t\cos\theta + h\sin\theta \le X_{(k)}\right\}$$
6. Environmental contours:
$$\mathcal{B} = \bigcap_{\theta \in [0,360)} \mathcal{B}(\theta)$$

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Identifying the boundary: Method I

- Calculate intersection points of neighboring tangent lines for angles θ and $\theta + \delta$

$$t \cos \theta + h \sin \theta = C(\theta)$$

$$t \cos(\theta + \delta) + h \sin(\theta + \delta) = C(\theta + \delta)$$

- The solutions give the coordinates of the intersection points, (t, h):

$$t = \frac{\sin(\theta + \delta) C(\theta) - \sin \theta \, C(\theta + \delta)}{\sin(\theta + \delta) \cos \theta - \sin \theta \cos(\theta + \delta)}$$

$$h = \frac{- \cos(\theta + \delta) C(\theta) + \cos \theta \, C(\theta + \delta)}{\sin(\theta + \delta) \cos \theta - \sin \theta \cos(\theta + \delta)}$$

- Having identified intersections points for a specified number of angles, the environmental contours are constructed by drawing lines between these points

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Identifying the boundary: Method II

- Let $\delta \to 0$ and find that the intersection points converge to

$$\binom{t}{h} = \begin{bmatrix} C(\theta) & -C'(\theta) \\ C'(\theta) & C(\theta) \end{bmatrix} \cdot \binom{\cos\theta}{\sin\theta}$$

  – $C'(\theta)$ denotes the derivative of $C(\theta)$

  – $C'(\theta)$ calculated numerically for all $\theta \in [0, 2\pi)$

- It is convenient to write the *C*-function on the following form, relative to a suitable point (t_c, h_c) in the middle of the cloud of simulated values (t, h), and with a suitable function D

$$C(\theta) = t_c \cos\theta + h_c \sin\theta + D(\theta)$$
$$C'(\theta) = -t_c \sin\theta + h_c \cos\theta + D'(\theta)$$

- which gives solutions on the form

$$\binom{t}{h} = \binom{t_c}{h_c} + \begin{bmatrix} D\theta) & -D'(\theta) \\ D'(\theta) & D(\theta) \end{bmatrix} \cdot \binom{\cos\theta}{\sin\theta}$$

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Identifying the boundary: Method III

- A variant of method II where a Fourier series expansion replaces interpolation to approximate the C-function
  – Improved smoothness of the environmental contours
- The C-function is periodic, repeating itself every 2π
  – It is therefore possible to approximate it by a Fourier expansion
- Fourier series of C(θ) is the infinite sum $C_F(\theta)$

$$C_F(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\theta) + b_n \sin(n\theta)]$$

With

$$a_n = \frac{1}{\pi}\int_0^{2\pi} C(\theta)\cos(n\theta)\,d\theta \qquad n \geq 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} C(\theta)\sin(n\theta)\,d\theta \qquad n \geq 1$$

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Method III (contd.)

- For some suitable integer N ≥ 1, the C-function can be approximated by the partial Fourier series $C_{F,\,N}$

$$C_{F,N}(\theta) = \frac{a_0}{2} + \sum_{n=1}^{N} [a_n \cos(n\theta) + b_n \sin(n\theta)]$$

- Since C(θ) is only known for certain θ–values, the integral for the Fourier coefficients must be estimated numerically
- Assuming that $C_{F,\,N}$ is a close approximation to the true C-function, a function defined for all $\theta \in [0, 2\pi]$ is produced
  – Straightforward to compute $C'_{F,N}$
  – Contours can be calculated using $C_{F,\,N}$ and $C'_{F,\,N}$
- Must specify number of tangents and number of Fourier terms
  – Must specify a reasonable value for N; depends on the number of tangents

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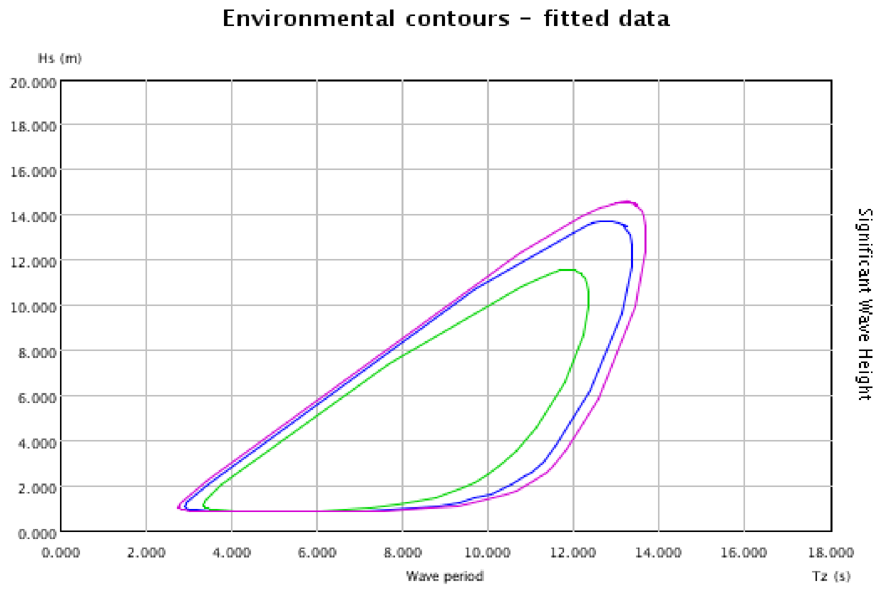
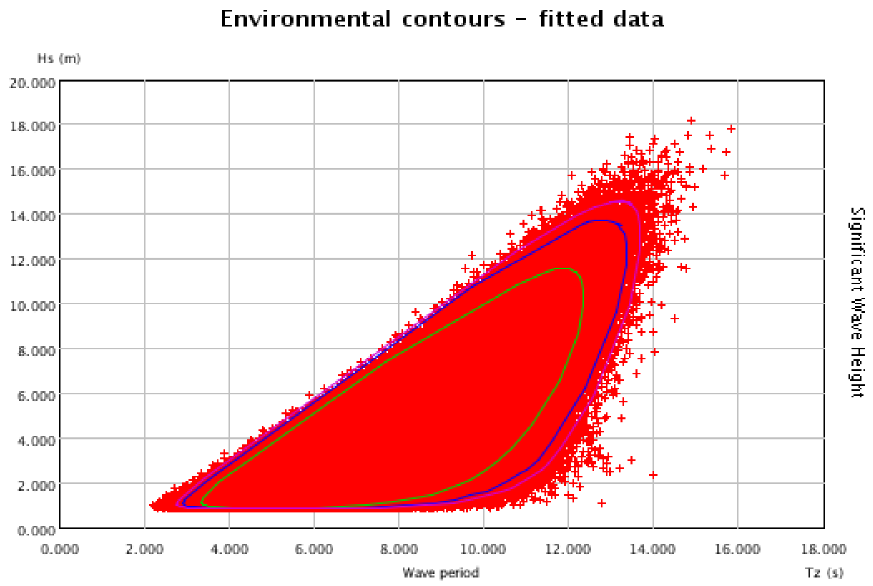
Case study

- Assume a conditional model for H and T: $f_{H,T}(h,t) = f_H(h) f_{T|H}(t|h)$
  - Marginal distribution for H, $f_H(h)$: 3-parameter Weibull distribution
  - Conditional distribution for T given H, $f_{T|H}(t|h)$: log-normal distribution
- Display contours for 1-, 10- and 25-year return periods based on n = 10 million samples (method I)





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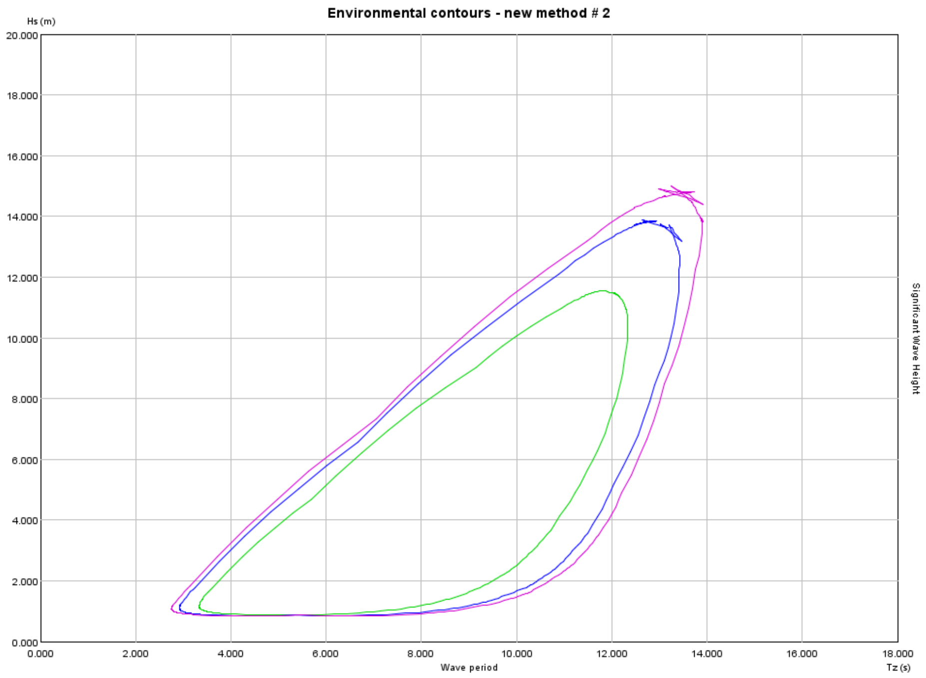
Case study (contd.)

Method II

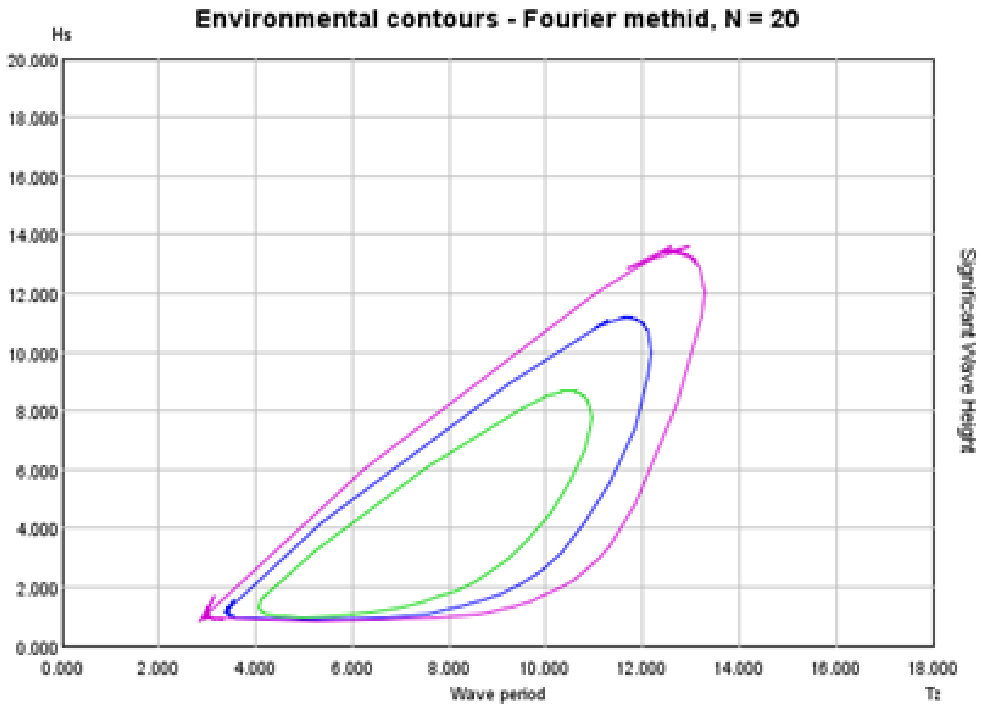
n = 1 million samples



Method III

n = 1 million samples

N = 20 Fourier terms



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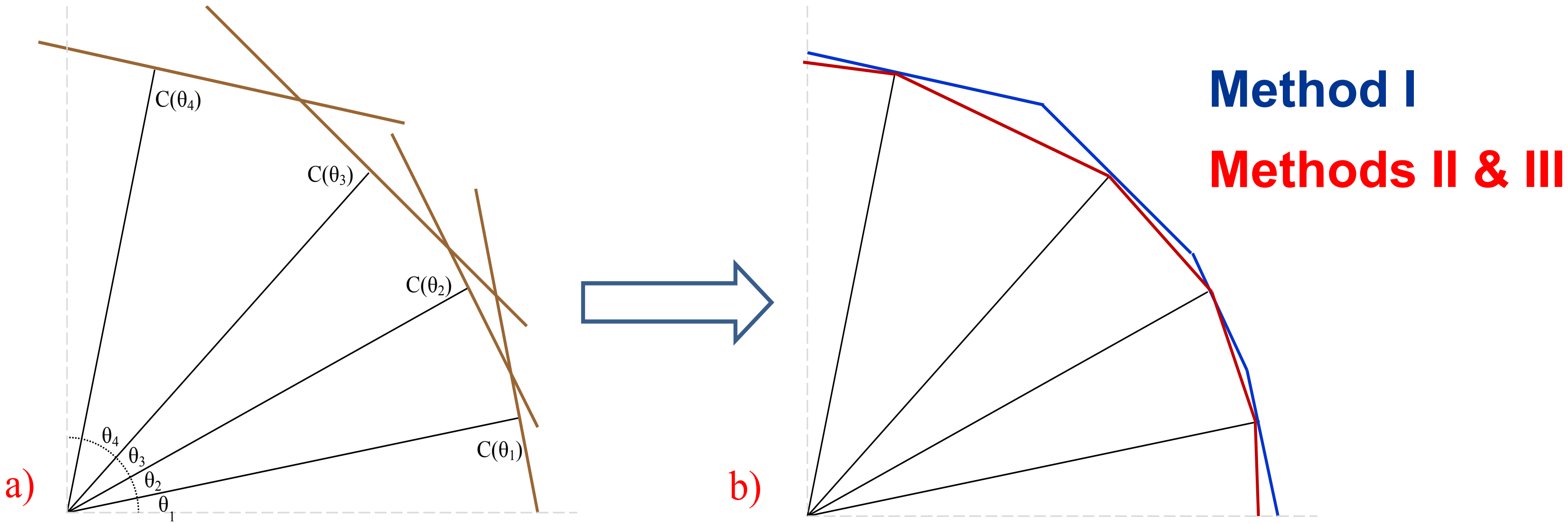
Method I vs. method II and method III

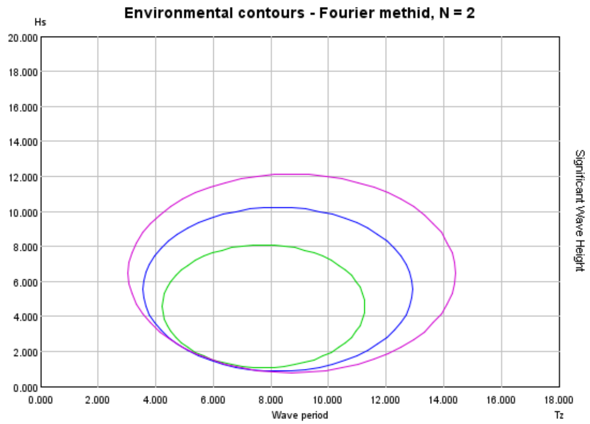
- Smoother contours with method II (and III)
- Contours obtained from method I more intuitive: Points along the contours are intersection points between two tangent lines – contour segments are segments of such tangents



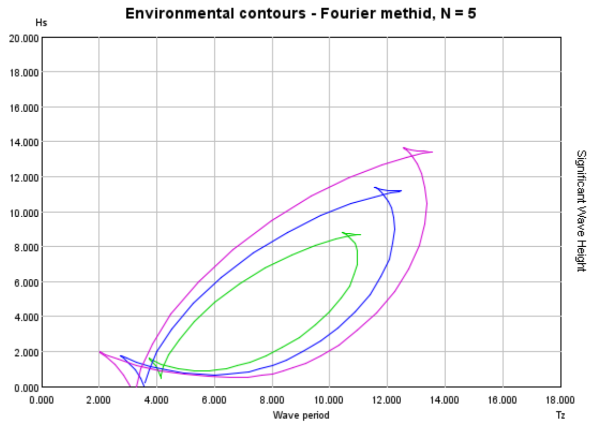
- The three methods will converge as the number of angles (tangents) increases

Method III: Sensitivity to number of Fourier-terms

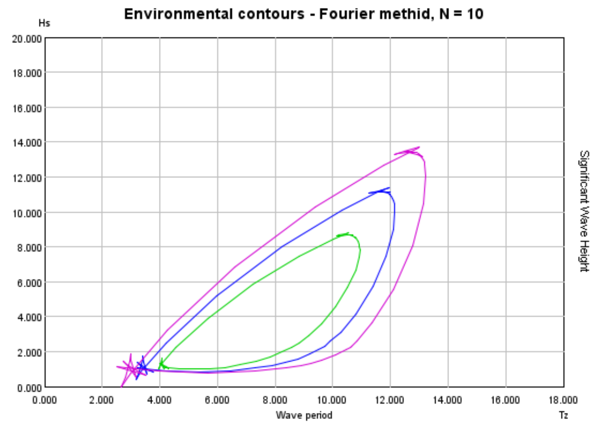
- Number of tangents = 60. Different number of Fourier-terms, N.



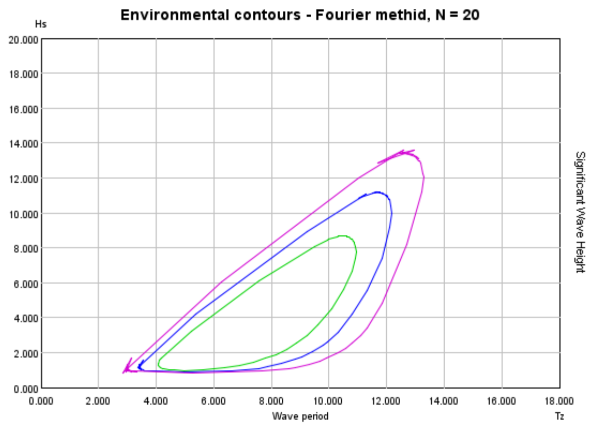
N = 2



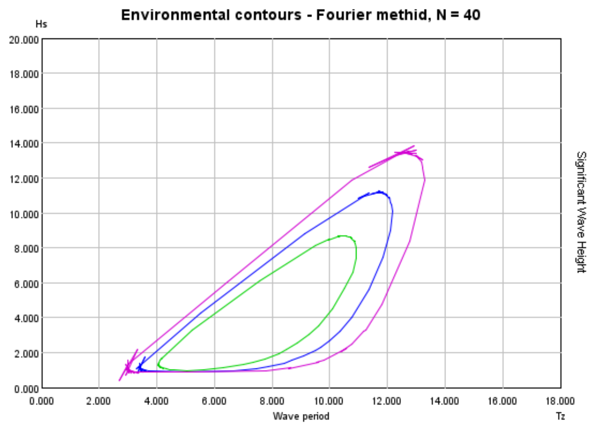
N = 5



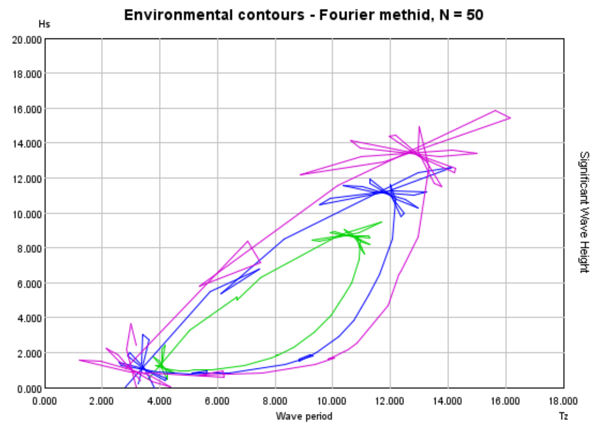
N = 10



N = 20



N = 40



N = 50

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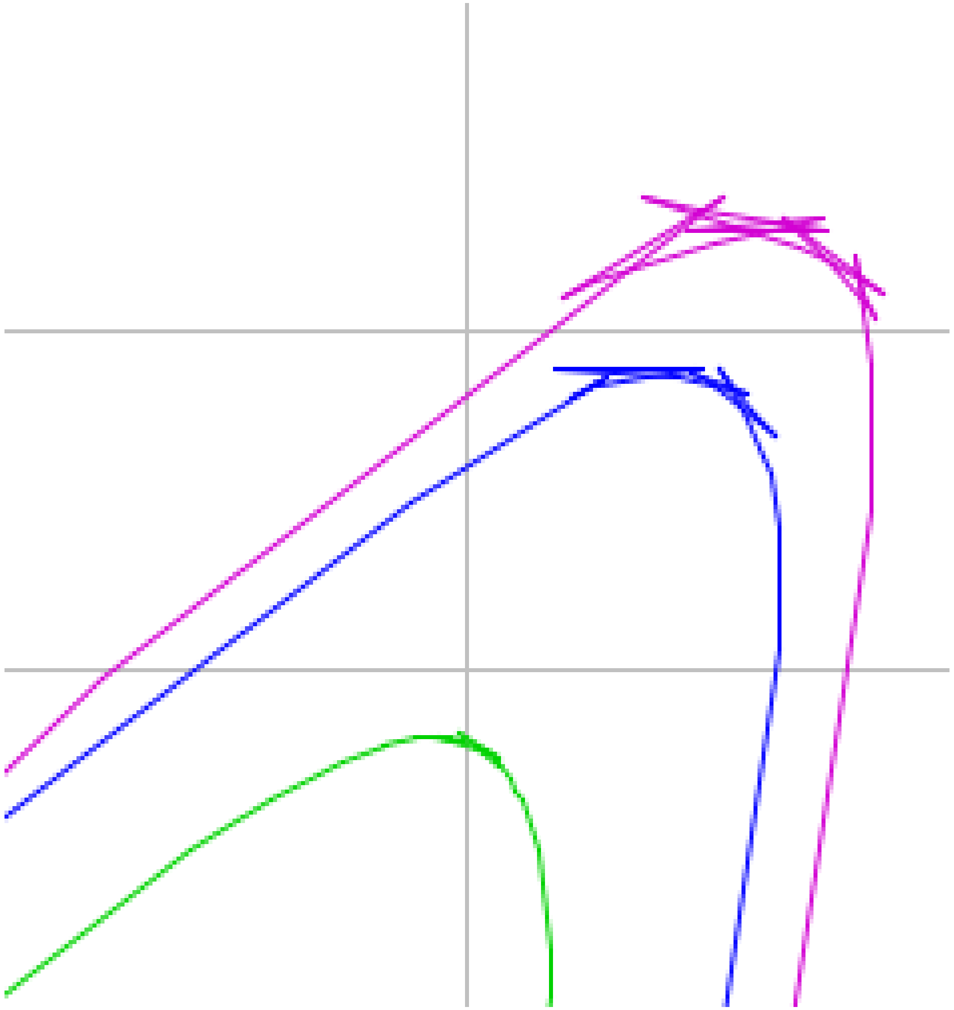
Sensitivity to sample size and angular resolution

- Irregular contours with small loops might occur due to
  - Too few samples
  - Too high angular resolution
- Increased sample size might be needed for very low failure probabilities (long return periods)
- May be solved by "importance sampling"



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Challenges using crude Monte Carlo simulation

- In typical applications P_f can be very small, i.e., less than 0.1%. Then a very large number of simulations is needed in order to obtain stable estimates
  - Processing the results in order to obtain the contours can be very time consuming
  - Storing a large number of simulations results in the computer memory can represent a challenge
  - Reducing the number of simulations yields noisy and unstable contours
- Most of the simulated points are close to the central area of the joint distribution, and thus very few results provide information about the contour area
- This situation can be improved by sensible “importance sampling”

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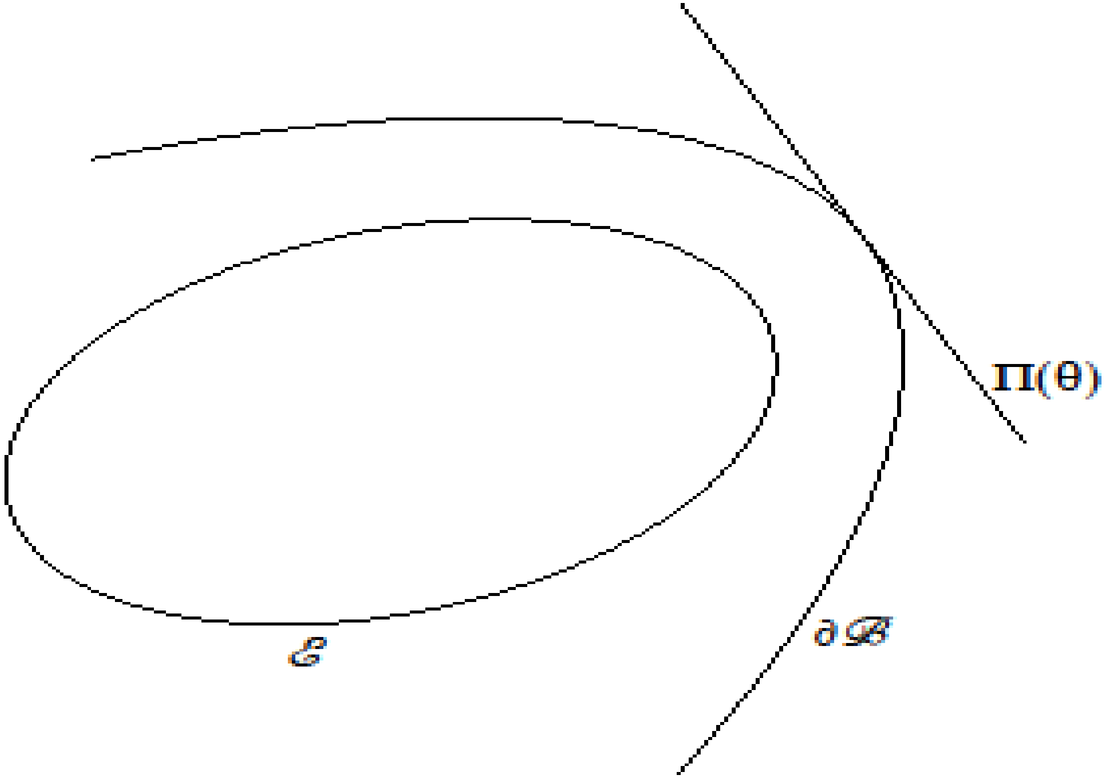
Improved Monte Carlo method

- Assume that we can find a subset $\mathcal{E}$ of the set $\mathcal{B}$. Then for any supporting hyperplane $\Pi(\theta)$ of $\mathcal{B}$ we have: $\mathcal{E} \subset \mathcal{B} \subset \Pi(\theta)^{-}$



- For all $(T_i, H_i) \in \mathcal{E}$ we know that $X_i(\theta) \leq C(\theta)$. Thus, we only need to store the *number of sampling points* inside $\mathcal{E}$, say e.

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Improved Monte Carlo method (contd.)

- Assume we have simulated $n = e + d$ observations from the joint distribution of (T, H), where d is the number of points outside $\mathcal{E}$:

$$(T_1, H_1), \dots, (T_d, H_d)$$

- For each of these d observations we calculate projections

$$X_i(\theta) = T_i \cos(\theta) + H_i \sin(\theta), \qquad i = 1, \dots, d$$

- The projections are then sorted in ascending order:

$$X_{(1)}(\theta) \leq X_{(2)}(\theta) \leq \cdots \leq X_{(d)}(\theta)$$

- Proceed by identifying an integer $k \leq d$ such that $\frac{k+e}{n} \approx 1 - P_f$
- Then $C(\theta)$ can be estimated by $\tilde{C}(\theta) = X_{(k)}(\theta)$

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Improved Monte Carlo method (contd.)

- Note that this corresponds to the adjusted exceedance probability of the reduced sample

$$P'_f = 1 - \frac{n\left(1 - P_f\right) - e}{d}$$

- Now, if $e > 0$ then $P'_f > P_f$ and the percentile estimates become more stable
- The set $\mathcal{E}$ can be found by way of the Rosenblatt transformation
  - Identifying a circular set $\mathcal{E}'$ within the environmental contours in standard normal space and find the set $\mathcal{E}$ by the inverse Rosenblatt transformation

$$\varepsilon = \psi^{-1}(\mathcal{E}') \subset \mathcal{B}$$

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Example: Bi-lognormal with Cov = 0.7

Crude Monte Carlo

- Using all 50000 points from a simulation run

$P_f = 0.001$

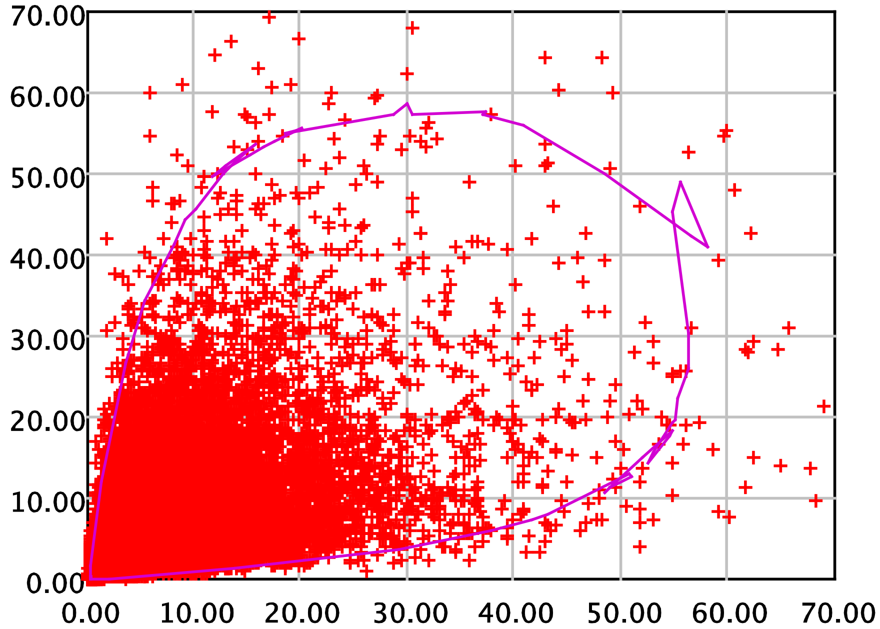
Importance sampling

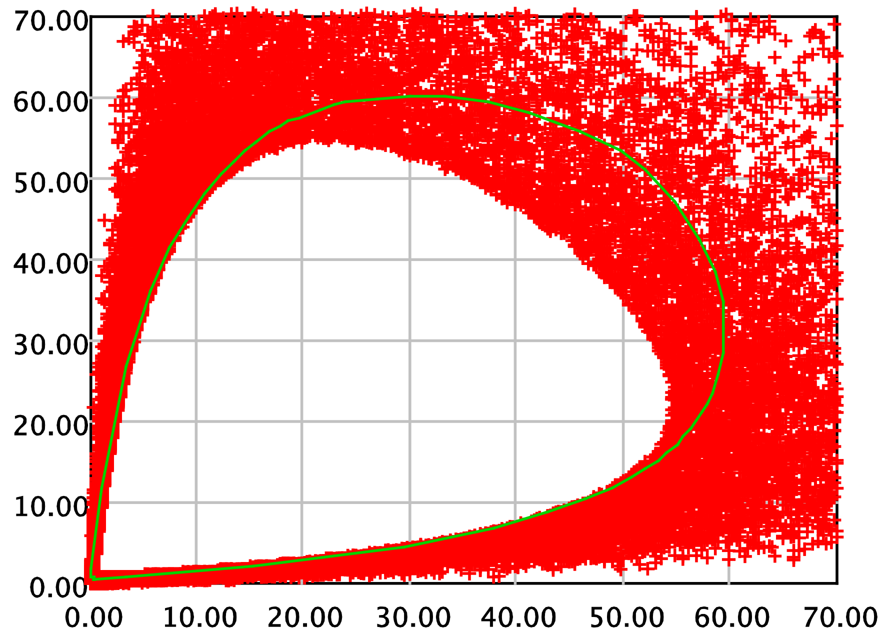
- Using a reduced set of 50000 points

$P_f = 0.001$ $d = 50\,000$

$e = 4\,452\,434$ $n = 4\,502\,434$

$P'_f = 0.09$





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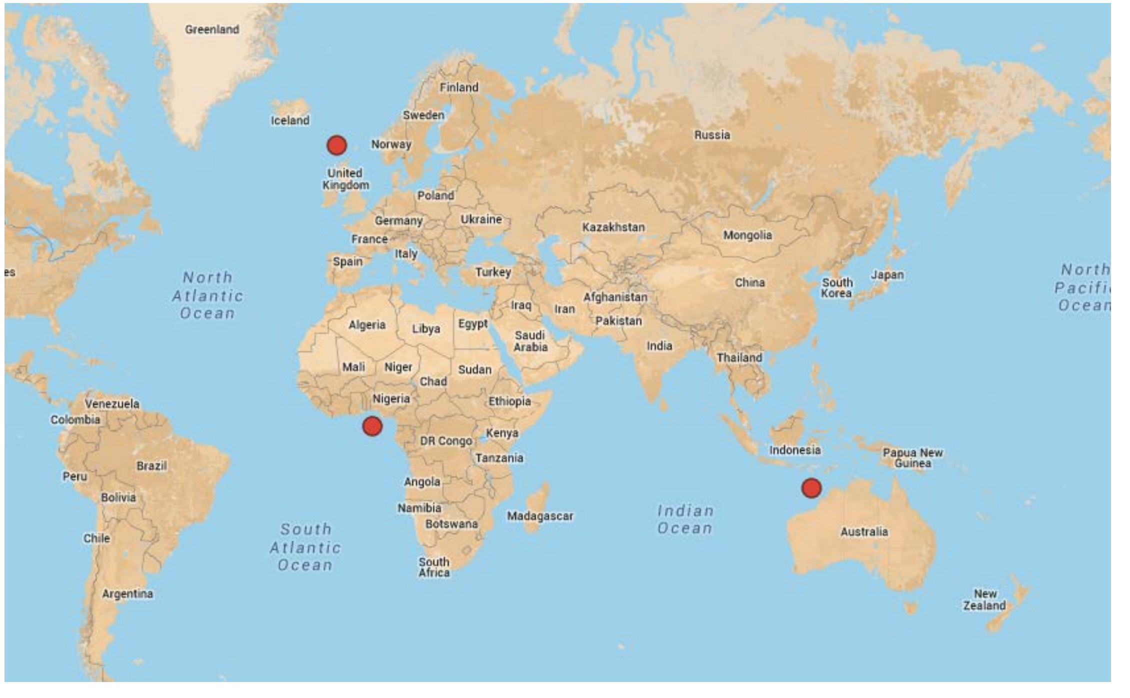
Comparison study

- Compare the traditional contours based on IFORM with the alternative contours based on Monte Carlo simulations
  – Contours based on met-ocean data for three specific locations
- NB: the two methods are essentially estimating **<u>different things</u>**
  – One is not merely an approximation of the other



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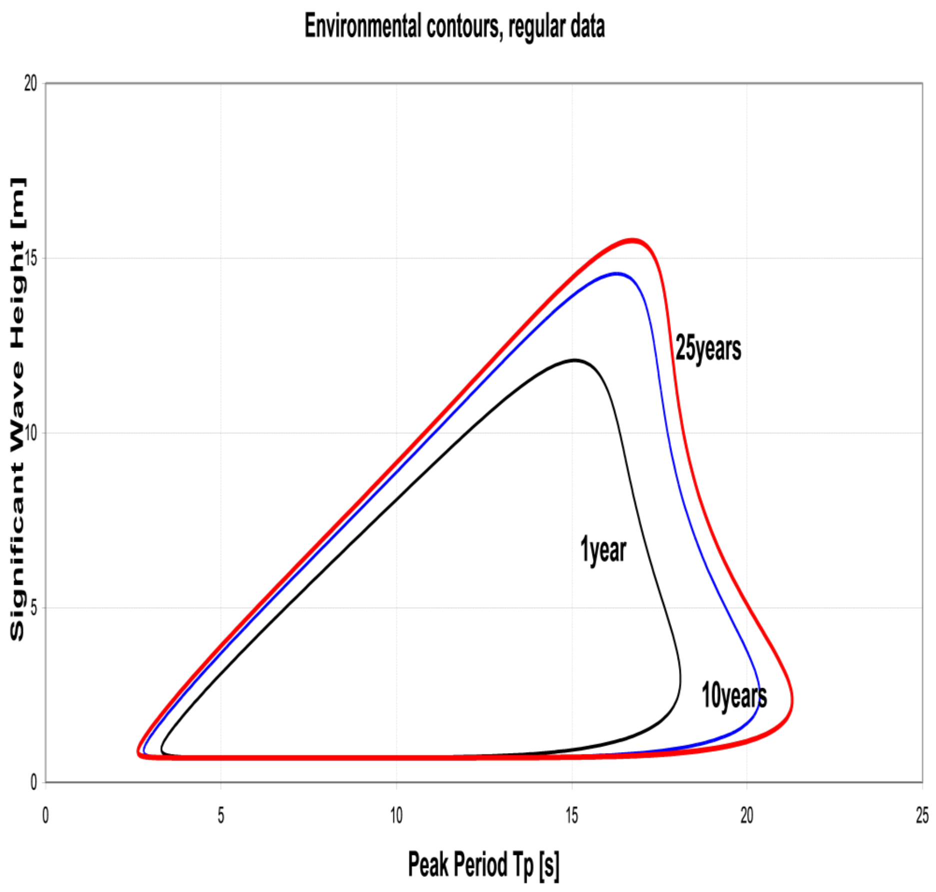
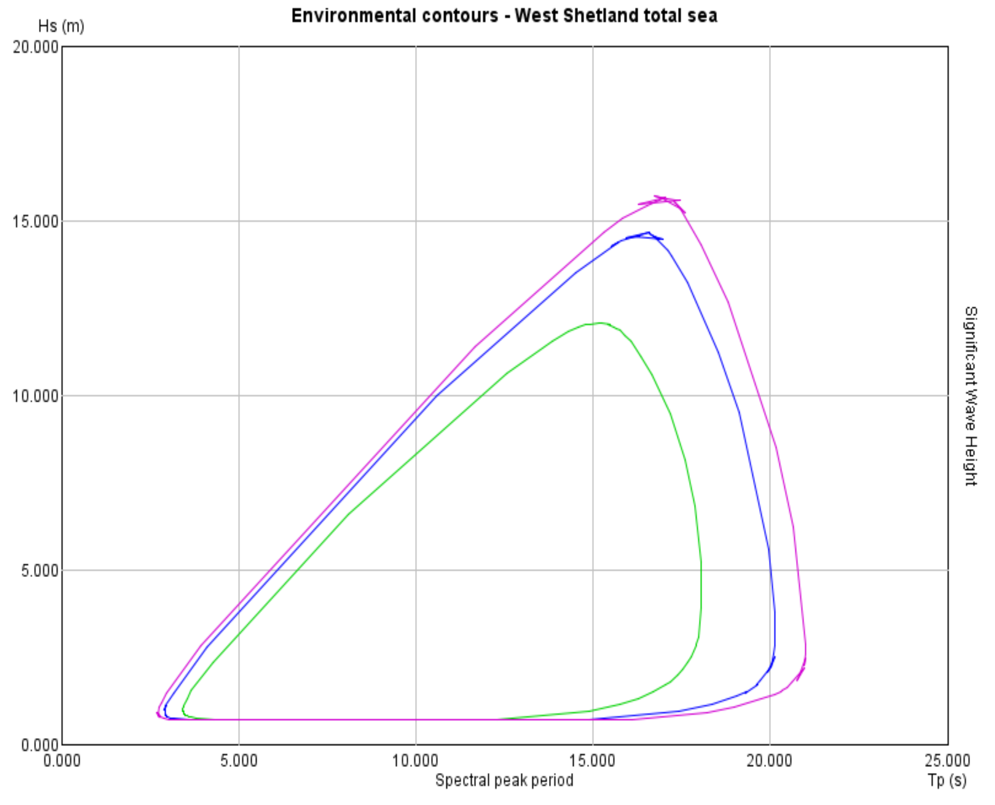
Estimated environmental contours 1

Location I: West Shetland total sea





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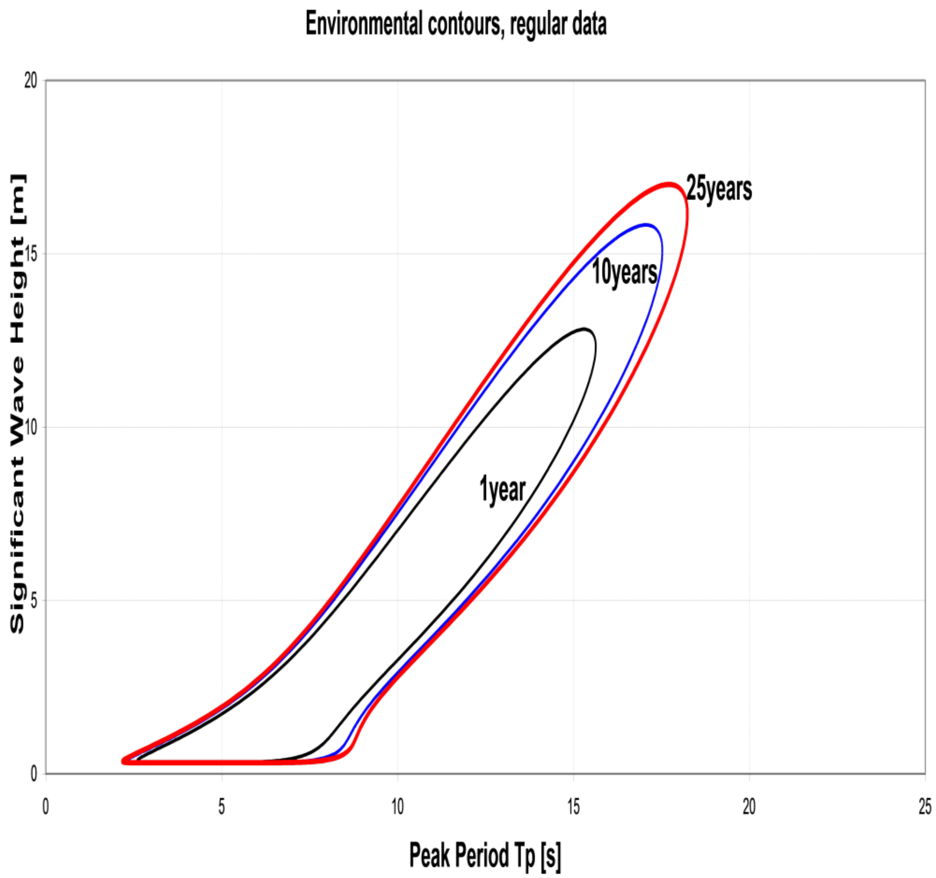
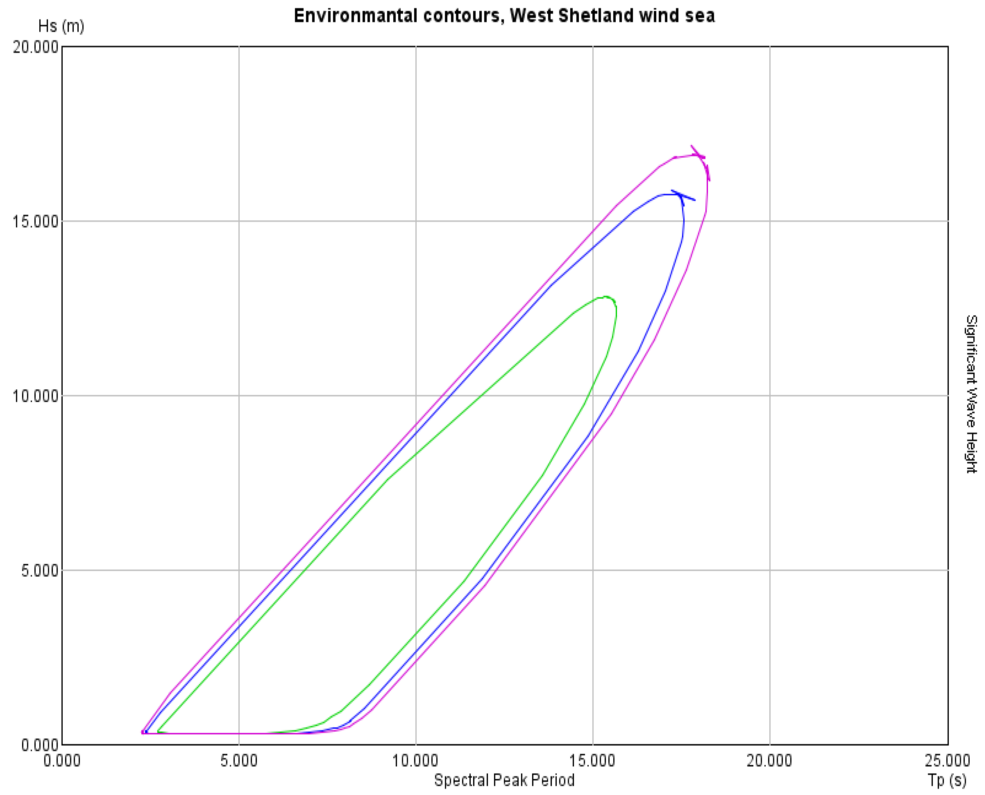
Estimated environmental contours 2

Location I: West Shetland wind sea





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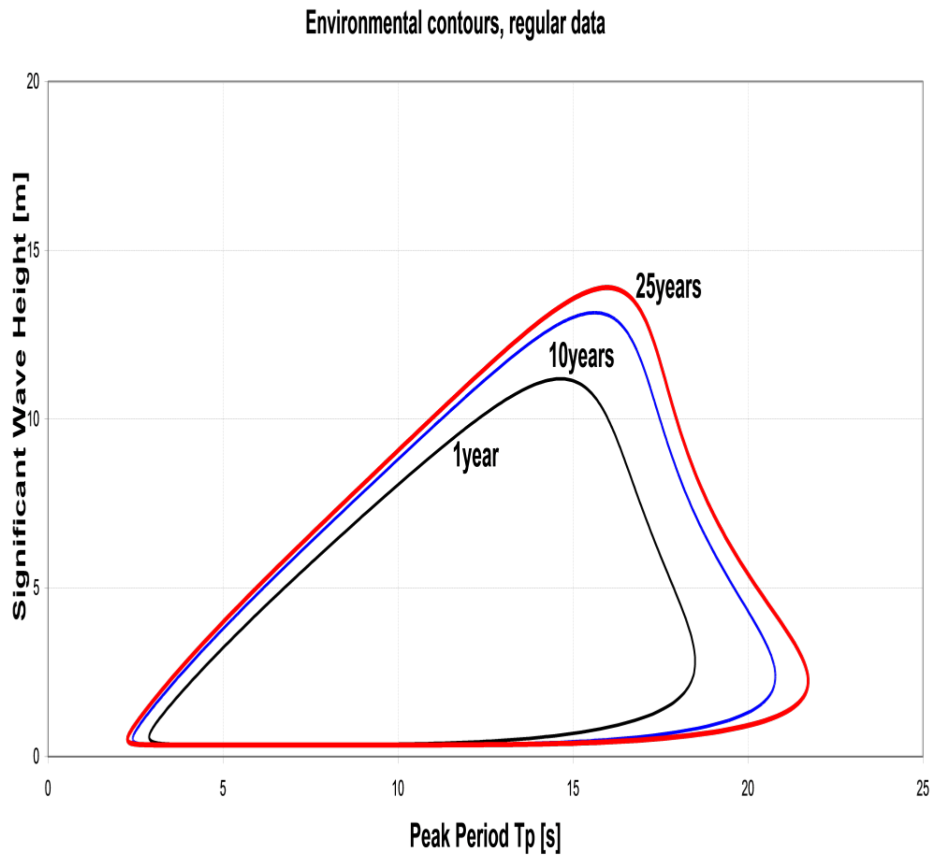
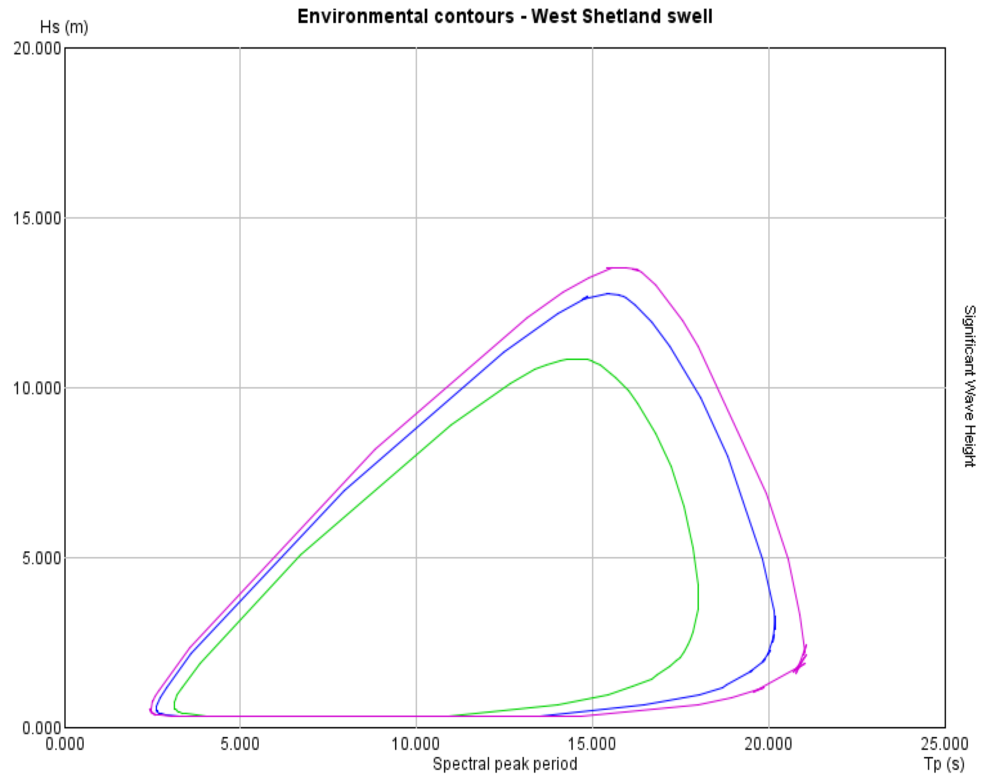
Estimated environmental contours 3

Location I: West Shetland swell





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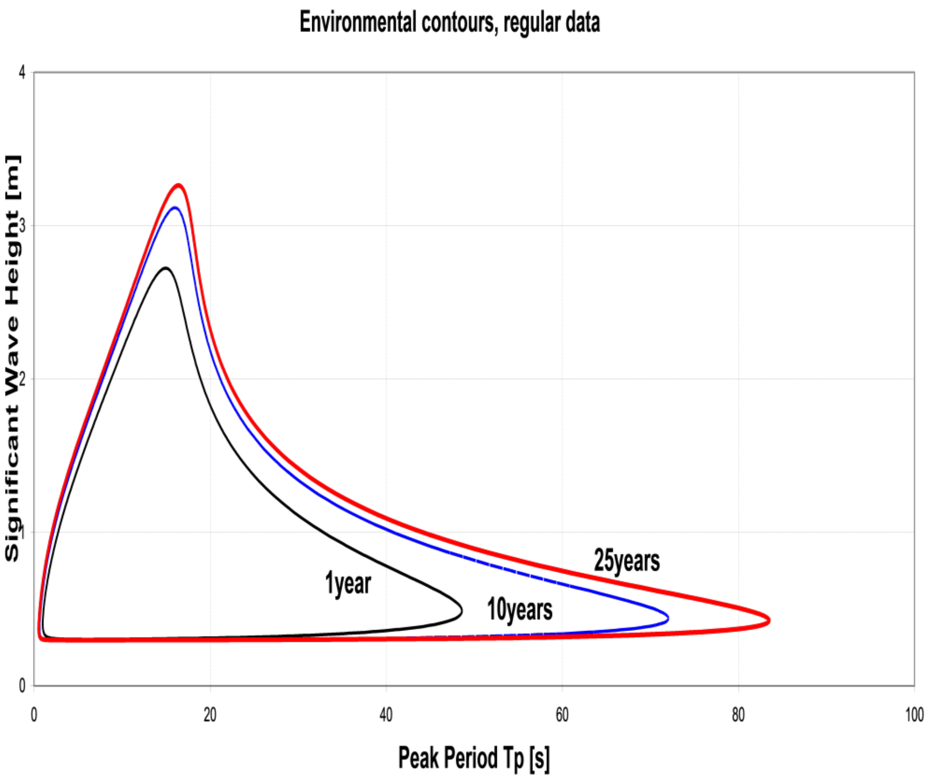
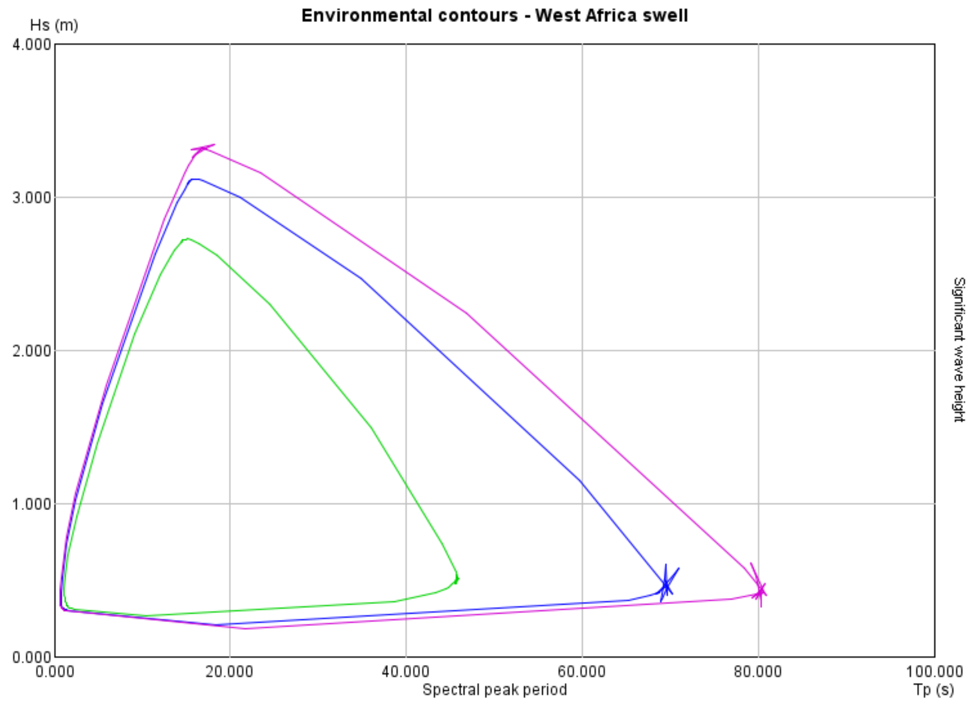
Estimated environmental contours 4

Location II: West Africa swell





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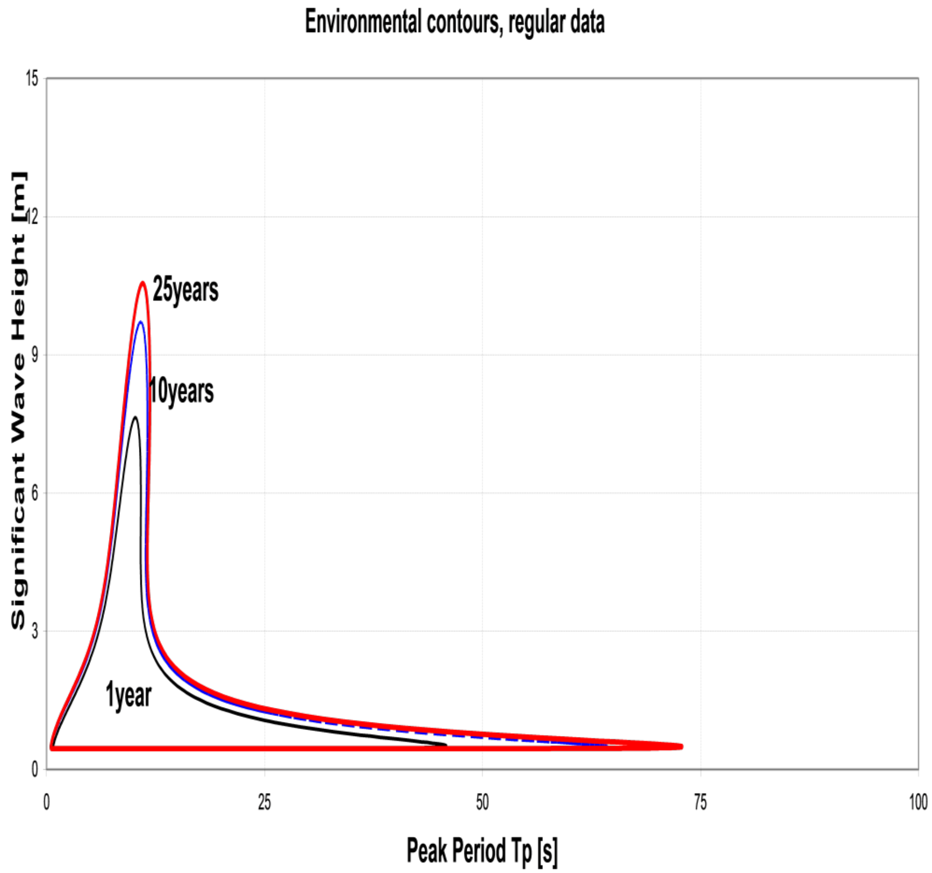
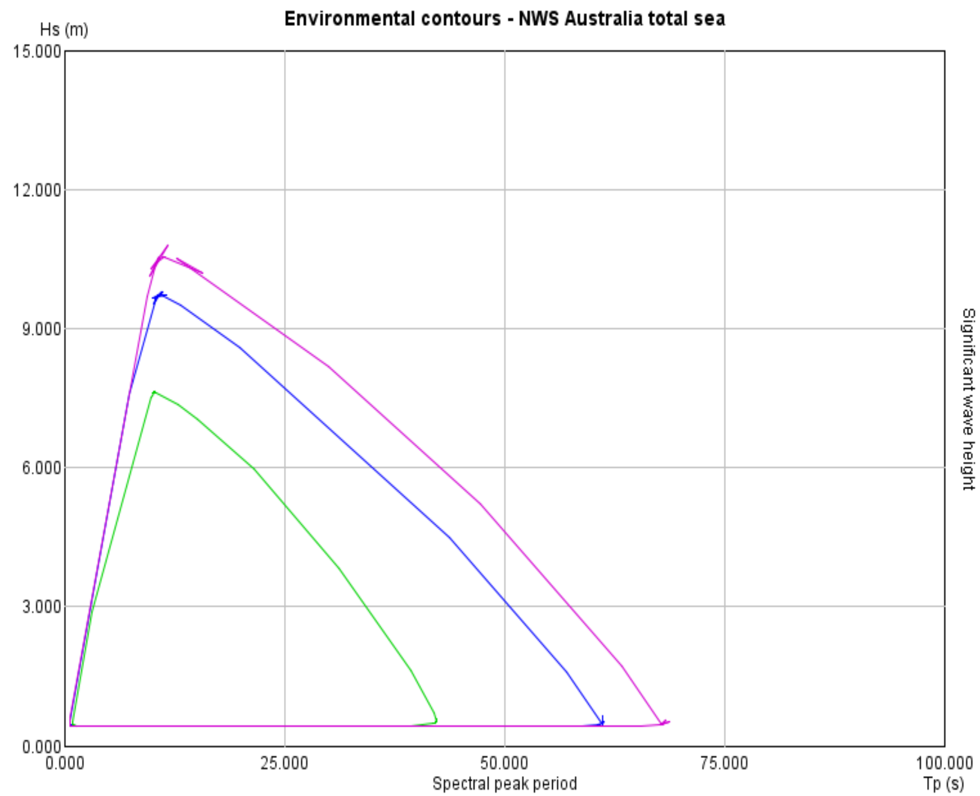
Estimated environmental contours 5

Location III: Northwest Australia total sea





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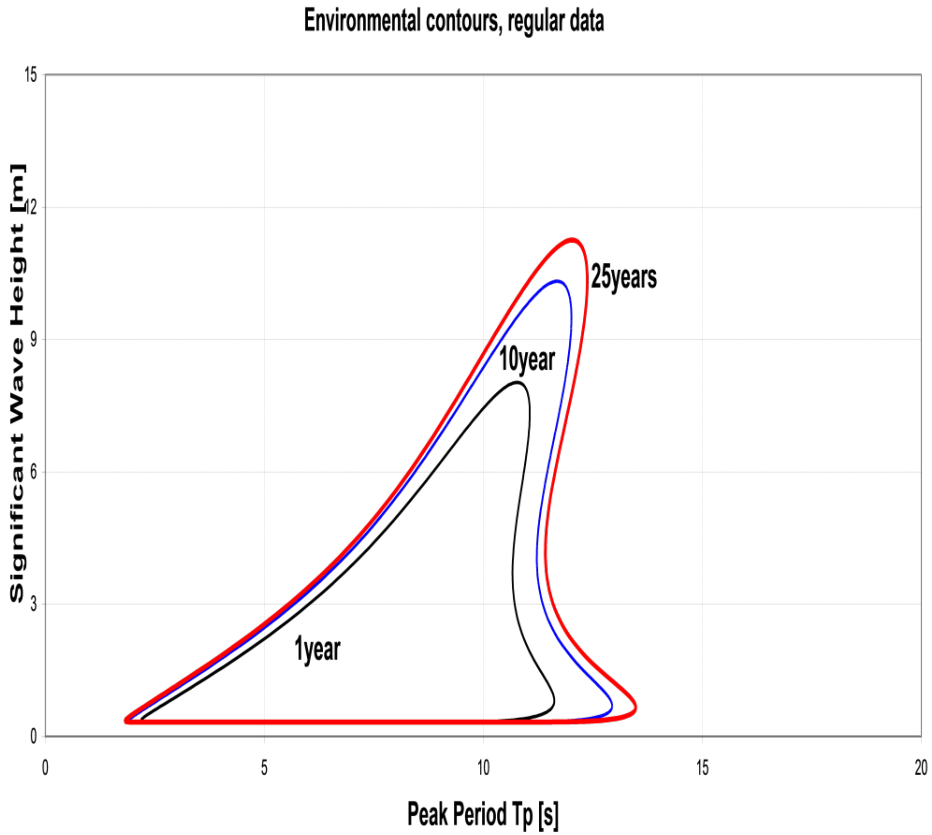
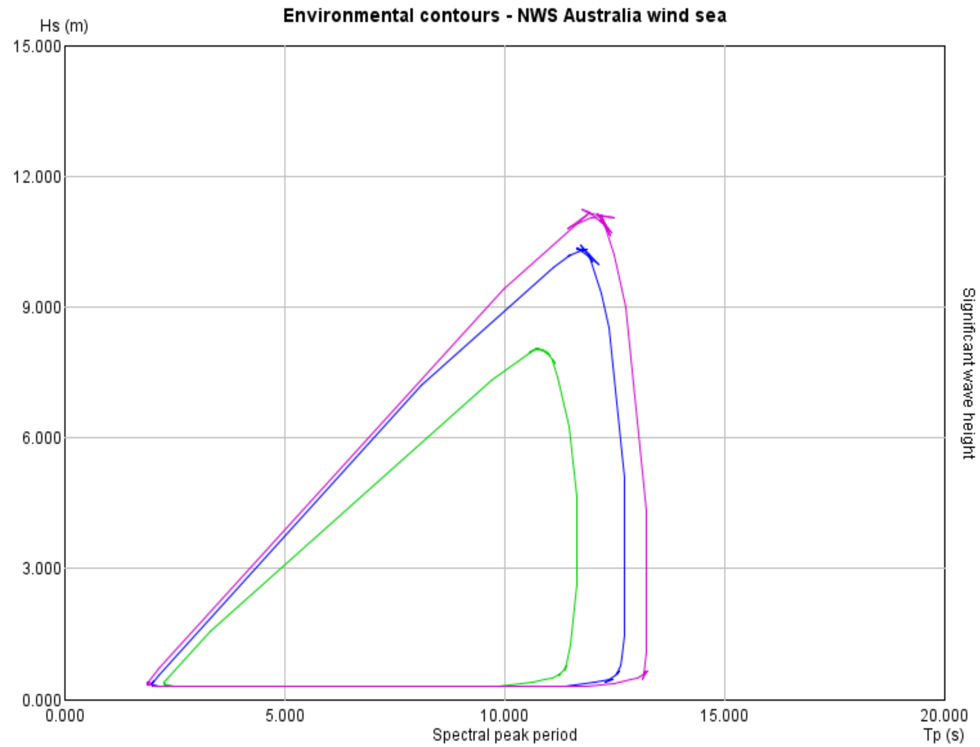
Estimated environmental contours 6

Location III: Northwest Australia wind sea





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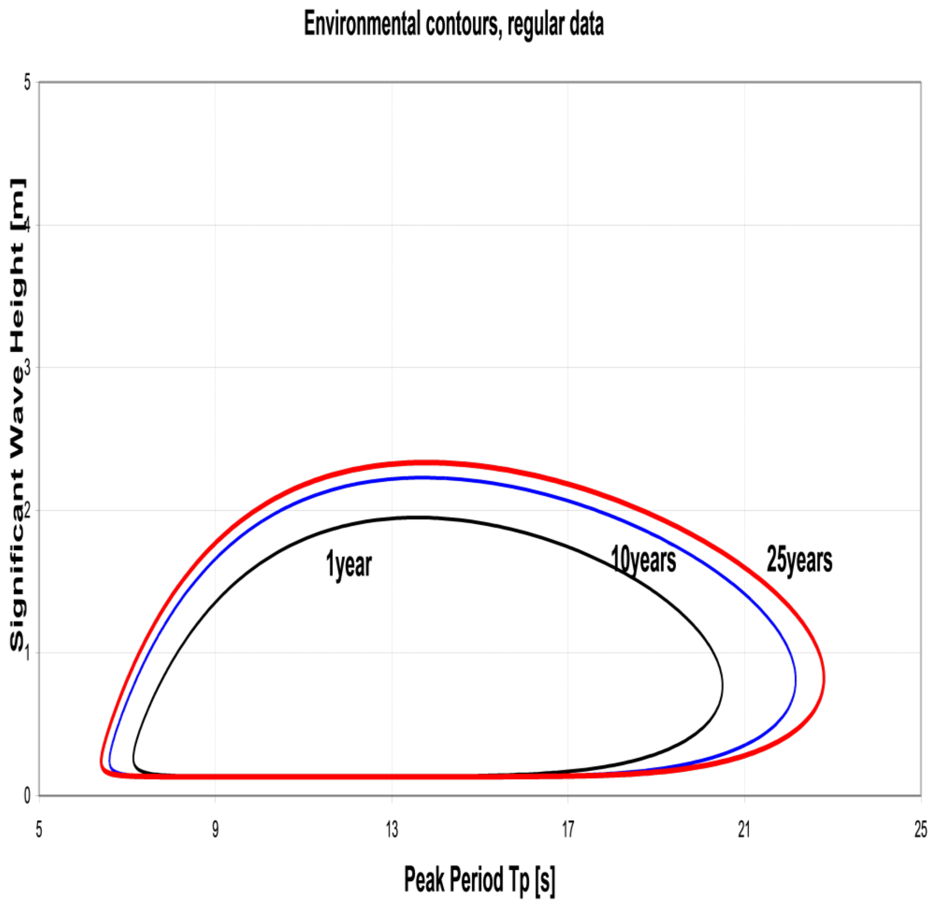
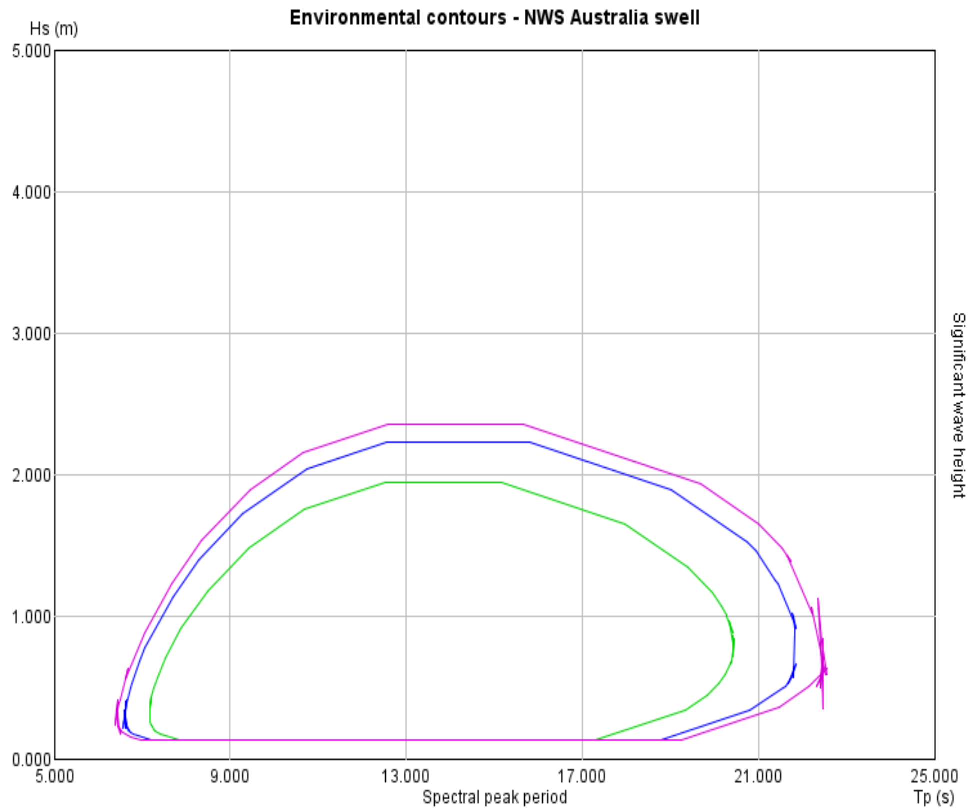
Estimated environmental contours 7

Location III: Northwest Australia swell





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Comparison of results

- For some data sets, the estimated contours look similar
  - E.g. West Shetland, NW Australia swell
- For other data sets, the contours look fundamentally different
  - E.g. West Africa swell, NW Australia total and wind sea
- The main reason for the differences is that the two methods estimate different features of the data
  - The traditional approach linearize the failure boundary in the transformed **U**-space and hence estimate tangent lines in the transformed space. These do not have well defined interpretation in the physical **X**-space
  - The sets defined by traditional contours need not be convex
  - The alternative approach estimates tangent lines with the required properties in the physical **X**-space. These tangent lines have the properties that the probability of being outside any of the tangent lines is approximately P_f
  - Due to the way they are defined, the alternative contours will always define a convex set with well defined tangent lines

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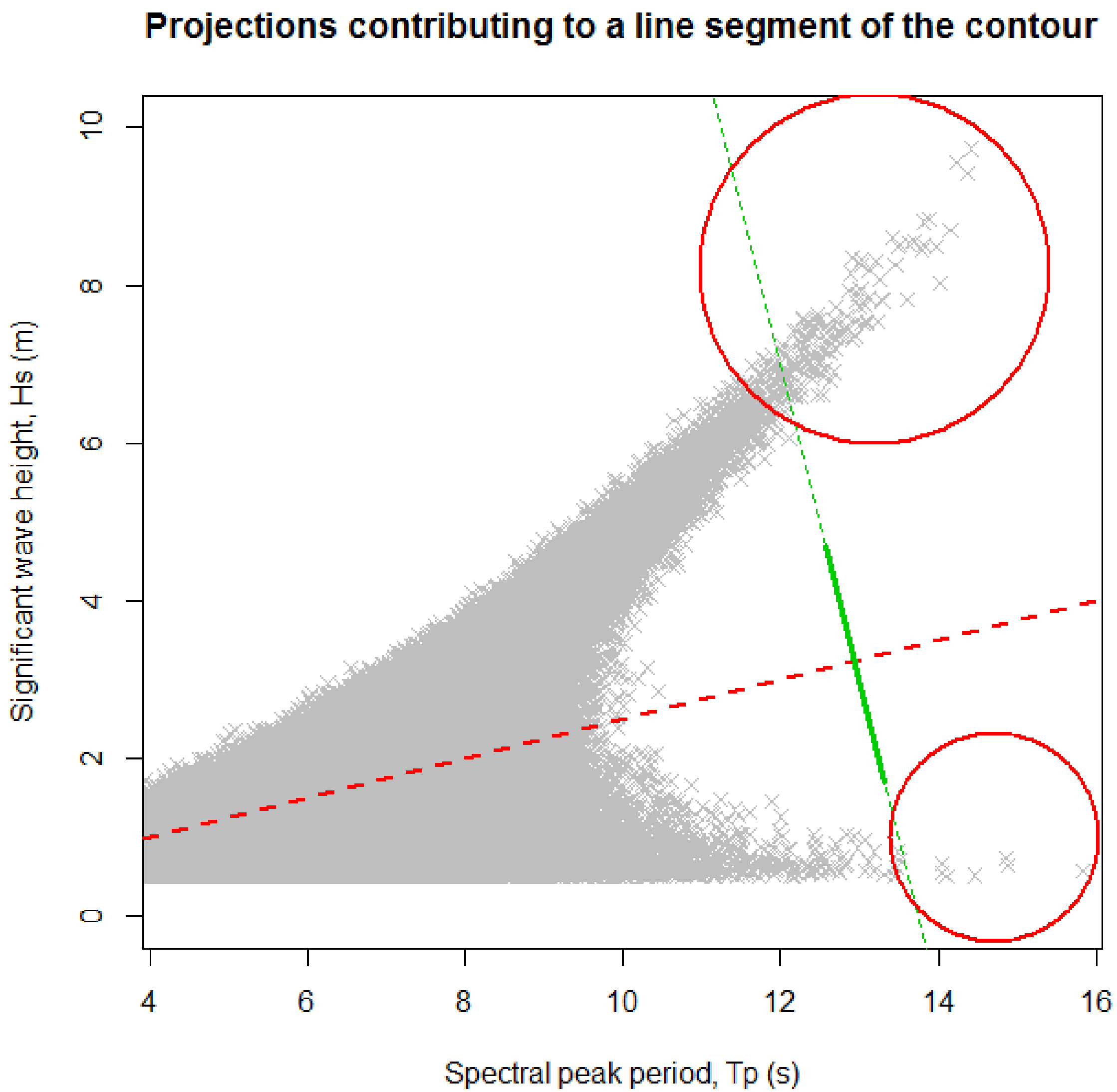
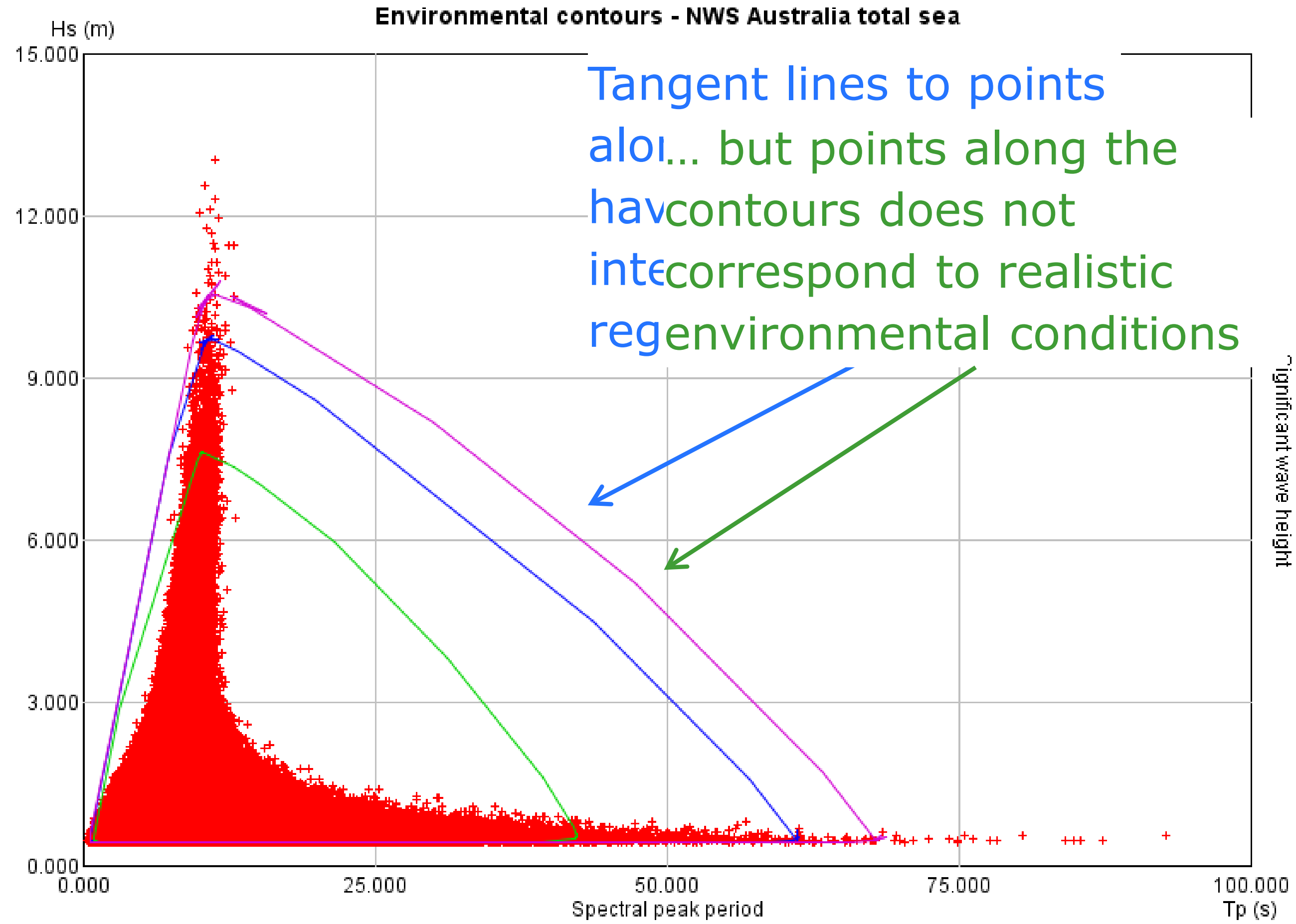
Explaining the differences



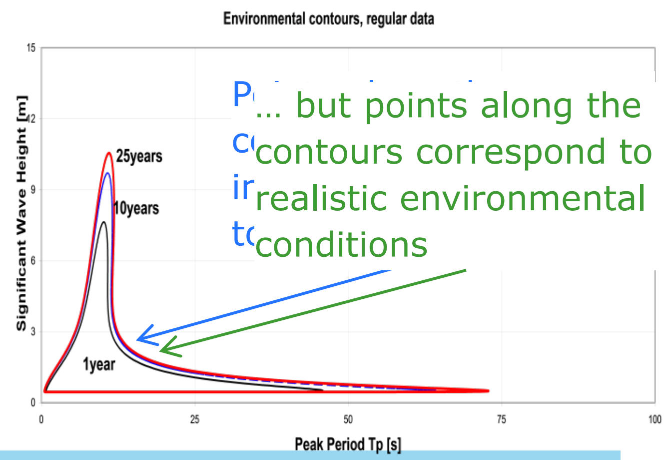
- All samples contribute to estimate the line segments in the alternative approach
- Contributions from remote samples in the parameter space tend to draw the segments outwards also in the parts of the contours where no such extreme sea states are observed

Estimated contours with scatter plot

Alternative contours



Traditional contours



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Pros and cons with the different methods

Traditional method

+ Points along the contours correspond to realistic environmental conditions, i.e. combinations of H_s and T_p with a certain probability of occurrence; contours follow the scatter plot
+ Proven method that is well established in the industry over many years. Also recommended by DNV-GL

– Points along the contours do not have a well defined interpretation with respect to probability
– Contours may be convex or non-convex and might not have the desired properties

Alternative method

+ Points along the contours have well defined interpretation with respect to probability
+ Will always estimate convex contours

– Points along the contours does not necessarily correspond to realistic environmental conditions
– New method that has yet to be proven in practice and to be accepted by the industry
– Some restrictions on the joint distributions that give contours with well-defined properties

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Summary and conclusions

- Environmental contours are useful in structural design of marine structures
  - Allows separation of the structural problem from the environmental description
  - Well established practice in the industry
- Traditional approach is based on transformation into standard normal space
- An alternative approach based on Monte Carlo simulations in the original space is proposed
  - Has some desired properties with regards to exceedance probabilities
  - Three different estimation methods are implemented
  - May be further enhanced by “importance sampling”

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Summary and conclusions

- The different approaches give very similar results in some well-behaved cases
- Fundamentally different results in other cases
  - These differences can be explained by the way the contours are defined
  - They are displaying essentially different features of the joint distributions
- Traditional contours follow the scatter plot of the data
- Alternative contours have well defined probability interpretation

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References

Original idea and the traditional approach:

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